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# String-Loop Effect in Low-Energy Effective Theory

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**Abstract** In this short article we are going to obtain the equations of motion from the low-energy effective action in the string cosmology. In the first time we consider the string-loop effect in the dilaton gravity and obtain the equations of motion, and obtain solution of them under some assumption for the specific potential.

**Keywords** String theory · String gas cosmology · Dilaton gravity

## 1 Introduction

The string theory is the most useful and promising candidate for a unified of the fundamental interactions including gravity. In the low-energy limit (low in comparison with the Planck mass) the string theory gives back the classical general relativity with the important differences among all versions of the string theory, which predict the existence of a scalar partner of the gravitational tensor which is called the dilaton field $\phi$. Low-energy effective string theory in context of the string gas cosmology studied in many papers such as [1–14]. String gas cosmology (Brandenberger–Vafa scenario) is one of the best models of the early universe. According to this model the early universe was small and compact and surrounded by hot and high dense string gas. In this scenario one deals with the background of ten dimensional dilaton gravity, with sources given by the string winding and momentum modes. It is found that the scale of the extra dimensions is stabilized at the self-dual radius, where many of the string modes become massless and the symmetry is enhanced [15]. A crucial aspect of these findings was the running of the dilaton to weak coupling, which was driven by the winding and momentum modes of the string. In this model corresponding effective action maybe written as the following,

$$S = \int d^{10}x\sqrt{-g}\big[e^{-2\phi}(R + 4(\nabla\phi)^2 + V) + \mathcal{L}_M\big], \tag{1}$$

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and the space–time metric takes the following form,

$$ds^2 = -dt^2 + \sum_{i=1}^{9} e^{2\lambda_i(t)} dx_i^2, \tag{2}$$

where $g$ is determinant of the background metric $g_{\mu\nu}$ and $\mathcal{L}_M$ is a lagrangian of some matter. The factor $e^{\phi}$ plays the role of coupling constant in the string theory. In that case the equations of motion are given by,

$$\begin{aligned} \sum_{i=1}^{9} \dot{\lambda}_i^2 - \dot{\psi}^2 &= e^{\psi+\sum\lambda}\rho, \\ \ddot{\lambda}_i - \dot{\psi}\dot{\lambda}_i &= \frac{1}{2} e^{\psi+\sum\lambda} p_i, \\ \ddot{\psi} - \sum_{i=1}^{9} \dot{\lambda}_i^2 &= -\frac{1}{2} e^{\psi+\sum\lambda}\rho. \end{aligned} \tag{3}$$

Damour and Polyakov expand the factor $e^{\phi}$ in powers of $g_s^2 = e^{2\phi}$, [16, 17], of the form,

$$B(\phi) = e^{-2\phi} + C_0 + C_1 e^{2\phi} + C_2 e^{4\phi} + \cdots. \tag{4}$$

The first term on the right-hand side of the expansion (4) is known [18–20]. The further terms represent the string-loop effect. The string-loop effects may generate matter couplings for the dilaton which allows the gravitational tensor to stay massless while contributing to macroscopic gravity in a way naturally compatible with existing experimental data. Under a certain assumption of universality of the dilaton coupling functions the cosmological evolution drives the dilaton towards values where it decouples from the matter. In this way the coupling to matter of the dilaton should be very small, but non zero. This provides a new motivation for improving the experimental tests of Einstein's Equivalence Principle [16, 17]. In the next section we consider the effect of string-loop in the equation of motion (3).

## 2 Equations of Motion

Dilaton-gravity comes from the low-energy effective action of the string theory. One can ignore contributions from the antisymmetric two-form and include a potential $V(\phi)$ for the dilaton. Also with respect to the string-loop effect the action of this system is described by the following,

$$S = \int dt \sqrt{-g_{00}}(-g^{00}) \left[ B(\psi) \left( \sum_{i=1}^{9} \dot{\lambda}_i^2 - \dot{\psi}^2 + V(\phi) \right) - F(\lambda_i, \beta\sqrt{-g_{00}}) \right], \tag{5}$$

where we have introduced a shifted dilaton,

$$\psi \equiv 2\phi - \sum_{i=1}^{9} \lambda_i, \tag{6}$$

and

$$e^{-\psi} = \sqrt{-g}e^{-2\phi}$$
$$e^{\sum_{i=1}^{9}\lambda_i} = \sqrt{-g}, \tag{7}$$

to simplify the obtained equations of motion. The function of the $\psi$ in (3), appearing as a common factor in front, is given by a series of the type,

$$B(\psi) = e^{-\sum\lambda_i}e^{-\psi} + C_0 + C_1 e^{\sum\lambda_i}e^{\psi} + \cdots, \tag{8}$$

which easily obtained by using (6) and (7) in (4). The first term of the right-hand side is the string tree level contribution. The further terms represent the string-loop effects. We kept just three terms of expansion (4). Also matter contribution to the action is represented by,

$$S_m = \int dt\sqrt{-g_{00}}F(\lambda_i, \beta\sqrt{-g_{00}}), \tag{9}$$

where $F$ is the free energy. Now in order to find the equation of motion one can consider variation of action (5). Solving the Euler-Lagrange equation with respect to $g_{00}, \lambda_i$ and $\psi$ yields the following equations of motion,

$$\sum_{i=1}^{9}\dot{\lambda}_i^2 - \dot{\psi}^2 = \frac{E}{B(\psi)} - V(\phi),$$
$$\ddot{\lambda}_i + \frac{B_{-2}(\psi)(\dot{\lambda}_i + \dot{\psi})^2 + 2C_0\dot{\psi}^2}{2B(\psi)} = \frac{P_i}{2B(\psi)} + \frac{B_{-1}(\psi)}{2B(\psi)}V(\phi) + \frac{1}{4}V'(\phi), \tag{10}$$
$$\ddot{\psi} + \frac{B_{-1}(\psi)}{2B(\psi)}(\dot{\lambda}_i + \dot{\psi})^2 - \frac{2C_0}{B(\psi)}(\dot{\lambda}_i + \dot{\psi})\dot{\psi} = -\frac{B_{-1}(\psi)}{2B(\psi)}V(\phi) - \frac{1}{4}V'(\phi),$$

where

$$B_{-2}(\psi) = -e^{-\sum\lambda_i}e^{-\psi} - C_0 + C_1 e^{\sum\lambda_i}e^{\psi},$$
$$B_{-1}(\psi) = -e^{-\sum\lambda_i}e^{-\psi} + C_0 + C_1 e^{\sum\lambda_i}e^{\psi}, \tag{11}$$

and

$$E = \rho e^{\sum\lambda_i}$$
$$= -2\frac{\delta S_m}{\delta g_{00}}$$
$$= F + \beta\frac{\partial F}{\partial\beta}, \tag{12}$$

is the total energy of the matter, and,

$$P_i = p_i e^{\sum\lambda_i}$$
$$= -\frac{\delta S_m}{\delta\lambda_i}$$
$$= -\beta\frac{\partial F}{\partial\lambda_i}, \tag{13}$$

is the total pressure in the $i$-th direction times the volume, which obtained by multiplying the total spatial volume $e^{\sum\lambda_i}$ with the energy density $\rho$ and pressure $p_i$, respectively. Also, in the equations of motion, we should note that $g_{00}$ is set to $-1$. Furthermore, dot denotes the time derivative and prime is the derivative with respect to $\phi$. One can solve equations of motion (10) numerically, however some assumption help us to have simplest equations, For example if we assume $C_0 = 0$ and $C_1 = 1$ the equations of motion reduces to,

$$\begin{aligned}\sum\dot{\lambda}_i^2 - \dot{\psi}^2 &= \frac{E}{2}\cosh^{-1}\left(\psi\sum\lambda_i\right) - V(\phi),\\ 2\ddot{\psi} + (\dot{\lambda}_i + \dot{\psi})^2\tanh\left(\psi\sum\lambda_i\right) &= -V(\phi)\tanh\left(\psi\sum\lambda_i\right) - \frac{V'(\phi)}{2}.\end{aligned} \tag{14}$$

Also one can use special potential for solving above equations such as brane world inflation [21],

$$V = V_0 e^{-\frac{\alpha\phi}{M_P}}. \tag{15}$$

In this model and under some assumption such as $\alpha = 2M_P$ one can obtain the following solution of the equation of motion,

$$\begin{aligned}x &= c_1 + \frac{c_2}{t} + c_3 t,\\ y &= ax,\end{aligned} \tag{16}$$

where we set $\psi = \ln x$ and $\lambda = \ln y$, also we assume that $x + y \gg 1$. The constants, $c_1$, $c_2$, $c_3$, and $a$ related to each other as following,

$$\begin{aligned}c_1 &= -\frac{E}{4A(1+a)} \pm \frac{1}{4A}\sqrt{\frac{E^2}{(1+a)^2} + \frac{8AV_0}{a}},\\ c_2 c_3^3 &= \frac{V_0}{2a(1+a)^2},\end{aligned} \tag{17}$$

where $\frac{\dot{x}}{x} \equiv \sqrt{A}$, so $A$ is another constant. All above constant may be determined by using initial conditions. Therefore we generalized the famous equations of motion of the string gas cosmology by using the idea of Damour and Polyakov.

## 3 Conclusion

In this paper we used the method of Refs. [16, 17] where the string-loop effect in the effective action of string considered. We generalized this method to the string cosmology whose effective action given by (1). We varied the action and obtained the equation of motion as relation (10). In this paper we consider just three terms of the expansion (4) so that one can check if we set $C_0 = C_1 = 0$, string-loop effect neglected, then the equations of motion reduces to the previous works.

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